

# Exponential Division – Axiomatic Definition

L'Architech (Gabriel Breton-Breault)

## ★ Axiomatic Definition of Exponential Division (Mathematician-Ready Version)

### Axiom 1 — Internal Direction Space

Let  $S$  be a compact geometric object equipped with a distinguished central point  $C$ . We associate to  $S$  an **internal direction space**:

$$\mathcal{D}(S) \subset T_C(S),$$

where  $T_C(S)$  denotes the tangent space at  $C$ . Elements of  $\mathcal{D}(S)$  are **internal directional vectors**, determined by the geometry of  $S$ .

### Axiom 2 — Primary Directions

We define a finite set of primary directions:

$$D_0 = \{d_1, d_2, \dots, d_n\} \subset \mathcal{D}(S),$$

where  $n$  depends on the geometry of  $S$ . Examples:

- sphere:  $n = 6$  (internal orthogonal directions),
- cube:  $n = 8$  (center  $\rightarrow$  vertices),
- tetrahedron:  $n = 4$ .

These are not coordinate axes; they are **intrinsic directional generators** of the object.

### Axiom 3 — Construction Rule for Secondary Directions

For each direction  $d \in D_k$ , we define a set of secondary directions:

$$\Delta(d) = \{f(d, d_i) \mid d_i \in D_0\},$$

where

$$f: \mathcal{D}(S) \times \mathcal{D}(S) \rightarrow \mathcal{D}(S)$$

is an internal directional operator given by:

$$f(d, d_i) = \frac{d + \lambda d_i}{\|d + \lambda d_i\|},$$

with fixed  $\lambda > 0$ .

Thus each secondary direction is a **normalized internal deviation** of the primary direction.

This axiom provides the **explicit geometric rule** that was missing in earlier formulations.

#### Axiom 4 — Exponential Division

Exponential Division is defined recursively by:

$$D_{k+1} = \bigcup_{d \in D_k} \Delta(d),$$

with:

$$|\Delta(d)| = |D_0|.$$

It follows that:

$$|D_k| = |D_0|^k.$$

This growth is **directional**, not volumetric.

#### Axiom 5 — Directional Density

We define the internal directional density:

$$\rho_k = \frac{|D_k|}{\text{Vol}(S)}.$$

The geometric volume of  $S$  remains constant, while:

$$\rho_k = \frac{|D_0|^k}{\text{Vol}(S)}$$

grows exponentially.

This distinguishes Exponential Division from:

- volumetric subdivision (density constant),
- fractal refinement (volume decreases),
- angular discretization (external directions).

#### Axiom 6 — Directional Saturation

Assume the existence of a minimal directional resolution  $\delta > 0$ . Directional saturation occurs when:

$$\rho_k \geq \rho_{\max} = \frac{1}{\delta}.$$

We define the maximal subdivision level:

$$k_{\max} = \left\lfloor \log_{|D_0|} \left( \frac{\text{Vol}(S)}{\delta} \right) \right\rfloor.$$

Beyond  $k_{\max}$ , no additional internal directions can be introduced.

#### ★ Summary (for mathematicians)

**Exponential Division is an internal directional operation defined on the tangent space of a compact geometric object. Starting from a finite set of primary directions, each direction generates a fixed number of secondary directions via a normalized internal operator. The total number of internal directions grows exponentially while the geometric volume remains constant. A minimal directional resolution imposes a saturation threshold, yielding a maximal subdivision level.**