

The Curvature of Spacetime in Potential Dimensions

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Introduction

Representing the curvature of spacetime is one of the hardest problems to visualize in physics. Two-dimensional representations give an intuitive picture of it, but they do not directly show a geometry acting simultaneously in all directions of space.

Directional physics proposes a different approach. It begins not with curvature itself, but with a redefinition of **dimension**.

In the model of potential dimensions, a dimension is not limited to a fixed axis such as X , Y or Z . It corresponds to a directional possibility accessible from a position. A position and the direction associated with it thus form the fundamental elements of the dimensional structure.

This conception makes it possible to represent the space around a point by a sphere. The center is the directional origin, while the surface of the sphere represents the set of directions accessible from that origin. This construction is already one of the foundations of Potential Dimensions.

When several spheres are considered successively, a spatial structure appears. Each surface represents a new level of positions and directions. Their size, spacing, and density may vary according to the local structure of space.

Around a mass, this succession therefore need not be regarded as a uniform series of independent surfaces. In the directional framework, local density also enters into the geometry. Dynamic Dimensions — Living Geometry of Space already develops the principle that a variation in directional density accompanies a modification of the internal directions.

Curvature can then be visualized in a different way.

In a three-dimensional representation, space is built from **successive spherical surfaces**. In a two-dimensional cross-section, these surfaces become **circles**. On each of these circles, corresponding positions can be identified. When these positions are followed successively, they form a line. If their orientation varies from one level to the next, this line becomes a **curve**.

In directional geometry, this curve represents a **curved dimension**.

Curvature is thus no longer considered solely as the deformation of a surface external to the dimension. It can emerge directly from the succession and transformation of the **position-directions that constitute the dimension itself.**

This construction also makes it possible to study several masses. In a system such as the Earth and the Moon, each body can be represented by a structure of spherical surfaces. The relations between these surfaces then produce a common geometry in the region separating the two bodies. A cross-section of this structure reveals the circles, the points of correspondence, and finally the directional curves.

The aim of this document is to build this representation step by step.

Chapter 1 will establish the definition of **potential dimensions** and specify their role in the description of space.

Chapter 2 will develop the three-dimensional representation based on **directional spheres** and their successive surfaces.

Chapter 3 will show how a cross-section of these spheres produces the **directional circles** that allow the geometry to be studied in two dimensions.

Chapter 4 will establish how the alignment and succession of corresponding positions on these circles produce **curves**, interpreted as curved dimensions.

Finally, **Chapter 5** will bring together spheres, circles, positions, directions, curves, and density in order to build **the geometric set of spacetime curvature within the framework of potential dimensions.**

Thus, curvature is no longer represented merely as the deformation of a geometric support. It becomes the manifestation of the organization and transformation of the directional dimensions that constitute space.

In directional physics, understanding curvature therefore begins with understanding what a dimension is.

Chapter 1 — Potential Dimensions

The curvature of spacetime in directional physics rests first of all on a particular definition of dimension. Before representing spheres, circles, or curves, we must determine what a dimension represents in this framework.

In potential dimensions, a dimension is not merely an abstract axis used to assign coordinates to a position. It represents a **directional possibility accessible from a given position**. Space can then be studied not only according to its extent, but also according to the number and organization of the directions it makes possible.

This definition is the starting point of the directional representation of curvature.

1.1. The dimension as position and direction

In this model, a dimension combines two fundamental properties:

a position and a direction.

The position indicates where the element under consideration is located in space. The direction indicates the possibility of movement associated with that position.

A directional dimension can therefore be represented conceptually by:

$$D_i = (P_i, \vec{d}_i)$$

where:

- P_i represents the position;
- $\vec{d}_i$ represents the direction associated with this position;
- D_i represents the corresponding directional dimension.

This definition makes it possible to stop regarding a dimension as necessarily rectilinear. A direction can change orientation from one position to another. A succession of directional dimensions can therefore form a straight, curved, or more complex trajectory.

1.2. All directions as potential dimensions

From a position P , several directions may be accessible.

The set of these possibilities constitutes the space of **potential dimensions**.

The idea is already formulated in Potential Dimensions: a dimension is defined there as an accessible direction, and the set of potential dimensions as the set of directions available from a point.

A fundamental distinction then appears between two concepts:

Potential dimension: a direction that the geometry makes possible.

Effective dimension: a direction actually used by a physical phenomenon.

In the formalism already established, effective dimensions therefore form a subset of potential dimensions:

$$N_{eff} \leq D_{pot}$$

Space may offer a very large number of directional possibilities without all of them being used simultaneously.

1.3. The sphere as a directional set

The sphere is a natural representation of this definition.

Consider a central point O . From this point, a direction can be drawn toward each distinguishable position on the surface of a sphere surrounding O .

The center then represents the **directional origin**, while the surface represents the set of orientations accessible from this origin. This interpretation is established in Potential Dimensions.

Thus, the sphere serves not only as a geometric shape. It becomes a representation of the **directional capacity of a position**.

The higher the resolution with which its surface can be distinguished, the greater the number of potential directions that can be represented.

1.4. Granularity and a finite number of dimensions

If space were perfectly continuous and no resolution limit existed, a surface could theoretically be subdivided indefinitely.

Directional physics instead introduces the hypothesis of a **minimal granularity**.

In Potential Dimensions, this construction uses the Planck length ℓ_P to define an elementary area:

$$a_{min} = \ell_P^2$$

and the number of potential directions associated with a sphere of radius R is then defined by:

$$D_{pot}(R) = \frac{4\pi R^2}{\ell_p^2}$$

In this model, a region of finite size and fixed minimal granularity therefore has a finite directional capacity.

This property becomes important for curvature: when the scale changes, the number of directional possibilities that the domain can contain changes as well.

1.5. A dimension is not necessarily rectilinear

The combination **position + direction** leads to an essential consequence.

Consider a succession of dimensions:

$$D_1 = (P_1, \vec{d}_1)$$

$$D_2 = (P_2, \vec{d}_2)$$

$$D_3 = (P_3, \vec{d}_3)$$

...

If the directional vectors keep the same orientation, their succession can produce a rectilinear structure.

But if:

$$\vec{d}_1 \neq \vec{d}_2 \neq \vec{d}_3$$

and this variation is gradual, the succession of positions produces a **curve**.

A dimension therefore need not be imagined as a straight axis that is subsequently deformed.

In this framework, **the possibility of curvature is already contained in the definition of dimension**, since the direction associated with positions can vary.

Dynamic Dimensions — Living Geometry of Space develops precisely this idea by linking the deformation of internal directions to the curvature of dimensions in directional geometry.

1.6. Directional density

The number of directions, however, is not enough to describe the directional space completely.

Their **density** must also be considered.

Dynamic Dimensions — Living Geometry of Space introduces a directional density and associates its variations with a contraction or stretching of the directional structure.

This means that two regions with comparable external geometry may, in the model, exhibit different directional organizations.

To represent the curvature of spacetime, three properties must therefore be considered together:

position + direction + density

The position indicates where a dimension is located.

The direction indicates its orientation.

The density describes the local organization of directional possibilities.

This combination will form the basis of the geometric representation developed in the following chapters.

1.7. Toward a representation of curvature

This definition now makes it possible to state the central principle of the document.

If every position in space has directional possibilities, and if these possibilities can vary gradually under the influence of the local structure, then the set of position-directions can form a curved geometry.

Curvature can thus be built from the succession of the dimensions themselves.

It remains, however, to find a representation that makes it possible to visualize simultaneously all the directions accessible around a position.

This representation will be the **sphere**.

The next chapter will show how a succession of directional spheres makes it possible to build a three-dimensional representation of the space around a mass, before this geometry is reduced to circles and then to curves in the following chapters.

Chapter 2 — Spheres

The previous chapter established the fundamental principle of potential dimensions: a dimension combines a **position** with a **direction**. To represent simultaneously all the directions accessible around a point, directional geometry uses the sphere.

This representation directly extends the principles developed in *Potential Dimensions* and *Dynamic Dimensions — Living Geometry of Space*. The sphere is not considered merely as a geometric object. It becomes a means of representing the directional capacity of space around a position.

2.1. The center as directional origin

Consider a sphere S with center O and radius R .

The center O is the directional origin. From this position, a direction can be drawn toward each distinguishable position on the surface.

A particular direction can be represented by:

$$\vec{d}_i = \overrightarrow{OP_i}$$

where P_i represents a position located on the surface of the sphere.

According to the definition of potential dimensions, each accessible position on this surface corresponds to a directional possibility from the center.

The sphere therefore makes it possible to represent simultaneously:

- an origin;
- a distance from this origin;
- a set of positions;
- a set of potential directions.

The dimension is no longer limited to a rectilinear axis. The directional possibilities completely surround the position under consideration.

2.2. The surface as a directional dimension

The spherical surface has a particular function in this representation.

All its points lie at a given distance from the center, but each corresponds to a different orientation.

The surface therefore constitutes a **directional level**.

This conception agrees with the principle developed in *Potential Dimensions*, where the surface of the sphere represents the set of orientations accessible from its center.

A single sphere, however, represents only one level of the structure.

To represent space beyond this surface, a second sphere must be introduced, then a third, and the construction continued.

One then obtains:

$$S_0, S_1, S_2, S_3, \dots, S_n$$

with successive radii:

$$R_0 < R_1 < R_2 < R_3 < \dots < R_n$$

Space can thus be represented as a succession of spherical surfaces.

2.3. The growth of spheres

The surface area of a sphere increases with its radius:

$$A(R) = 4\pi R^2$$

A larger sphere therefore has a larger surface.

In the model of potential dimensions, this increase also has a directional interpretation. If a minimal granularity is maintained, a larger surface allows more positions, and thus more potential directions, to be distinguished.

In *Potential Dimensions*, this directional capacity is expressed by:

$$D_{pot}(R) = \frac{4\pi R^2}{\ell_p^2}$$

when the minimal area is defined from the Planck length.

The radius of the sphere thus directly influences its directional capacity.

The larger the sphere, the greater the number of potential dimensions it can contain.

The size of space and its directional richness thus become linked.

2.4. A succession of spheres

Now consider several spheres with the same center:

$$S(R_0), S(R_1), S(R_2), \dots, S(R_n)$$

Each surface represents a new possible position according to the depth considered, as well as a new set of potential directions.

For a given orientation, it then becomes possible to identify a corresponding point on each of the surfaces:

$$\begin{aligned}P_0 &\in S_0 \\P_1 &\in S_1 \\P_2 &\in S_2 \\&\dots\end{aligned}$$

These points are not independent. They may belong to a single directional continuity crossing the different surfaces.

This means that the complete geometry is not made up only of spheres.

It is also made up of the **directional relations between their different positions**.

It is this property that will later allow curved dimensions to appear.

2.5. Depth as dimensional structure

The succession of spheres also introduces the notion of depth.

A first sphere represents one level. A second sphere represents a different level. By repeating the operation, a three-dimensional structure appears.

Depth is therefore not external to the surfaces. It is produced by their succession.

This progression can be represented by:

$$S_0 \rightarrow S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow \dots$$

Each transition corresponds to a new position in the depth of the structure.

Directional geometry thus obtains a representation in which **surface and depth together participate in the construction of space**.

2.6. The density of spheres

However, a perfectly regular succession of spheres is not enough to represent the curvature proposed in directional physics.

Directional density must also be considered.

In *Dynamic Dimensions — Living Geometry of Space*, directional density plays a role in the contraction and stretching of the directional structure.

A denser region should therefore not be represented in exactly the same way as a less dense region.

The geometry must simultaneously account for:

- the radius of the sphere;
- its surface;
- the number of potential directions;
- the directional density;
- the relation to neighboring surfaces.

Thus, two successive levels are not characterized by their radius alone. Their relation also depends on the directional state of the space separating them.

This property becomes essential when a mass is introduced.

2.7. Spheres around a mass

Let us now consider a spherical body such as a planet or a star.

A first directional surface can be associated with its outer region. Another surface is then considered at the next interval, then another, and so on.

The mass thus finds itself surrounded by a succession of spheres:

$$S_0 \rightarrow S_1 \rightarrow S_2 \rightarrow \cdots \rightarrow S_n$$

The resulting representation has a fundamental property: **it exists in all directions around the mass.**

There is no privileged depression downward.

There is no sheet on which the mass must be placed.

The geometry is entirely volumetric.

Each orientation around the center crosses a succession of surfaces and directional positions.

In the framework developed in *Dynamic Dimensions — Living Geometry of Space*, the presence of matter is associated with a modification of the directional structure and density. The succession of spheres must therefore reflect this evolution.

2.8. Two masses and two spherical structures

The construction can then be extended to two masses.

Consider two bodies *A* and *B*.

The first has its own succession of surfaces:

$$S_{A,0}, S_{A,1}, S_{A,2}, \dots$$

The second also has its own:

$$S_{B,0}, S_{B,1}, S_{B,2}, \dots$$

As the spheres grow, the two structures progressively occupy the region between the masses.

The surfaces coming from the two systems can then meet and form a common geometric structure.

The Earth–Moon system can be represented in this way.

The Earth has a family of spherical surfaces. The Moon also has its own family of surfaces. Between the two bodies, the surfaces associated with the different positions of space join together in a single directional geometry.

Curvature is then no longer represented as two hollows drawn on a sheet.

It appears in **the three-dimensional organization of the surfaces and directions linking the two masses**.

2.9. Contact points and dimensional continuity

The relations between the different surfaces become particularly important.

Each point of a sphere represents a position-direction. When a continuity exists with the next surface, a new point can be associated with it.

We obtain:

$$P_0 \rightarrow P_1 \rightarrow P_2 \rightarrow P_3 \rightarrow \dots$$

By repeating this construction for all possible orientations, the entire space can be viewed as a set of continuities between surfaces.

If these points remain perfectly aligned, the corresponding dimension retains a rectilinear form.

If their relative positions change progressively under the effect of the directional geometry and density, their alignment changes as well.

A curvature then begins to appear.

However, this curvature remains difficult to observe directly when all the three-dimensional spheres are represented simultaneously.

It therefore becomes useful to temporarily reduce this three-dimensional geometry to a two-dimensional representation.

A planar cross-section through a sphere yields a circle.

A succession of spheres thus becomes a succession of circles.

These circles will make it possible to isolate the corresponding points, observe their change of position, and finally visually reconstruct the curve constituting a directional dimension.

This transformation will be the subject of **Chapter 3: Circles**.

Chapter 3 — Circles

The previous chapter presented spheres as a three-dimensional representation of the directional structure of space. Each spherical surface constitutes a level for representing a set of positions and directions around a center.

To observe the relation between these surfaces more simply and to understand how a curved dimension can appear, it is possible to take a **two-dimensional cross-section** of the three-dimensional construction.

In this cross-section, the spherical surfaces become **circles**.

The circle is therefore not a structure independent of the sphere. It represents the two-dimensional manifestation of a geometry that remains fundamentally three-dimensional.

3.1. From the sphere to the circle

Consider a sphere S cut by a plane passing through its center.

The resulting section forms a circle C .

This transformation can be represented by:

$$S \rightarrow C$$

The sphere allows us to consider the directions around the center throughout the whole three-dimensional space. The circle selects only the directions belonging to the cutting plane.

This reduction makes it possible to observe the structure without altering its fundamental principle.

The circle must therefore always be interpreted as **the cross-section of a spherical surface**.

3.2. A succession of spheres becomes a succession of circles

Let us now consider the successive surfaces developed in the previous chapter:

$$S_0, S_1, S_2, S_3, \dots, S_n$$

If the same plane cuts through this entire construction, each sphere produces a circle:

$$C_0, C_1, C_2, C_3, \dots, C_n$$

with:

$$S_i \rightarrow C_i$$

The three-dimensional construction can thus be studied in a much simpler two-dimensional form.

Each circle represents a particular level of the directional structure.

The succession of circles represents the succession of spherical surfaces observed in a single plane.

3.3. The circle as a closed directional structure

The circle has an essential property for directional geometry: it is **finite but closed on itself**.

Moving along its circumference, a position can change continuously while remaining on the same structure. After a complete circuit, the displacement returns to its point of origin.

The direction thus evolves continuously along the circumference without requiring a terminal boundary.

This illustrates an important property of directional dimensions:

a dimension can be finite without necessarily having an endpoint.

The continuity of the dimension therefore does not depend on its straightness.

It depends on the continuity of the positions and directions that constitute it.

3.4. Position-directions on a circle

Let us now take a position P_i on a circle C_i .

This position also has a local direction $\vec{d}_i$.

A directional dimension can thus be represented by:

$$D_i = (P_i, \vec{d}_i)$$

The position indicates where the directional element lies on the circle.

The direction indicates how the continuity can proceed from this position.

Each circle therefore contains a set of position-directions.

If the circle is subdivided according to a minimal granularity, the distinguishable positions on its circumference also become directionally distinguishable.

The circle thus becomes a two-dimensional representation of the directional capacity of the level considered.

3.5. Corresponding points between circles

Consider several successive circles:

$$C_0, C_1, C_2, C_3, \dots, C_n$$

The same directional continuity can be followed by selecting a corresponding position on each:

$$P_0 \in C_0$$

$$P_1 \in C_1$$

$$P_2 \in C_2$$

$$P_3 \in C_3$$

and so on.

We then obtain:

$$P_0 \rightarrow P_1 \rightarrow P_2 \rightarrow P_3 \rightarrow \dots \rightarrow P_n$$

These points represent different successive positions of the same directional continuity.

It is this succession that will make it possible to pass from a set of circles to an identifiable dimension.

3.6. Variation in the size of the circles

The circles need not have identical radii.

Since they represent different spherical surfaces of the three-dimensional construction, their size depends on the geometry represented.

We can thus obtain:

$$R_0 < R_1 < R_2 < R_3$$

or, in a more complex structure, a succession in which the radius increases and then decreases.

This variation becomes particularly important when we seek to represent the space between two masses.

In the directional model, the evolution of the size of the circles makes it possible to represent the successive transformations of the geometry linking different regions.

3.7. Directional density

The size of the circles is not, however, sufficient to represent the structure completely.

The **directional density** must also be taken into account.

Two regions can have similar geometries while exhibiting different densities.

For each level i , the representation must therefore consider simultaneously:

$$C_i, P_i, \rho_i$$

where:

- C_i represents the circle associated with the level;

- P_i represents the position-direction considered;
- ρ_i represents the local directional density.

The representation by circles thus becomes a means of studying not only geometric shapes, but also the way position-directions are distributed among the different levels.

3.8. Two masses in the 2D representation

Let us now consider two masses, such as the Earth and the Moon.

In the three-dimensional construction, each is associated with spherical surfaces. These surfaces belong to the same space, and their relations contribute to the geometry of the region between the two bodies.

When a plane cuts through this construction, the surfaces appear as circles.

The two-dimensional representation can then be built as a succession of circles progressively linking the two regions.

The size of the circles can evolve from the scale associated with the Earth to that associated with the Moon.

It is nevertheless essential to retain the three-dimensional interpretation:

the circles visible in the drawing are not space itself. They represent cross-sections of the spherical surfaces constituting the three-dimensional construction.

The 2D representation thus becomes a window through which to observe a particular portion of the complete geometry.

3.9. The circle as the contact point of the representation

In the three-dimensional construction, it is the **surfaces of the spheres** and their relations that determine the structure.

When this geometry is reduced to a plane, the relations between the surfaces appear through the circles.

The corresponding points on these circles then become particularly important.

Each point represents a position belonging to a three-dimensional surface, but observed in the cutting plane.

Thus, a series of points:

$$P_0, P_1, P_2, P_3, \dots, P_n$$

is not simply an arbitrary set of coordinates.

Each point retains three-dimensional information behind it: **the point belongs to the cross-section of a spherical surface and represents a position-direction of that surface.**

3.10. From circles to the curve

We can now observe the successive points without displaying the entire circumference of each circle.

If the points are connected:

$$P_0 \rightarrow P_1 \rightarrow P_2 \rightarrow P_3 \rightarrow \cdots \rightarrow P_n$$

a line appears.

If the positions evolve without directional change, this line can remain straight.

However, if the position-directions change progressively from one circle to the next, the line also progressively changes orientation.

It becomes a **curve**.

The complete construction can then be summarized as follows:

spheres → circles → position-directions → curve

The spheres represent the three-dimensional structure.

The circles represent a two-dimensional cross-section of this structure.

The points isolate a particular directional continuity.

The curve finally reveals the form taken by this continuity through space.

This last step is fundamental, because it will allow us to establish a direct geometric representation of what directional physics defines as a **curved dimension**.

The next chapter will therefore develop **Chapter 4 — Curves**.

Chapter 4 — Curves

The previous chapters progressively built the geometric representation of potential dimensions. Spheres make it possible to represent the set of directions around a position in three-dimensional space. Circles constitute a two-dimensional cross-section of these

surfaces. It now becomes possible to isolate corresponding positions on each circle and connect them.

This operation brings out the **curve**.

In directional physics, this curve has a particular meaning. It does not simply represent the trajectory of an object crossing an already curved space. It can represent the very form of a **directional dimension**, built from the continuity of a succession of positions and directions.

Curvature thus becomes a geometric property of the organization of potential dimensions.

4.1. From position to directional continuity

Consider a succession of circles:

$$C_0, C_1, C_2, C_3, \dots, C_n$$

On each of these circles, a corresponding position-direction is identified:

$$D_i = (P_i, \vec{d}_i)$$

We then obtain:

$$P_0, P_1, P_2, P_3, \dots, P_n$$

Taken separately, these points describe different positions of the structure.

But when a directional continuity exists between these positions, they can be connected:

$$P_0 \rightarrow P_1 \rightarrow P_2 \rightarrow P_3 \rightarrow \dots \rightarrow P_n$$

A geometric line appears.

This line represents the continuity of a direction across several successive levels of the structure.

4.2. The straight line as a special case

If the successive points keep exactly the same orientation, the resulting line is straight.

This situation can be represented by a constant direction:

$$\vec{d}_{i+1} = \vec{d}_i$$

In this case, the different positions remain aligned along a single orientation.

The straight line thus becomes a **special case** of the directional dimension: the one in which the direction does not change over the succession of positions.

This distinction is important.

In the directional model, a dimension need not be defined as straight. Straightness simply represents a particular state of its geometry.

4.3. The appearance of curvature

Now consider a different situation.

The direction changes slightly between two successive positions:

$$\vec{d}_{i+1} \neq \vec{d}_i$$

If this change continues progressively:

$$\vec{d}_0 \rightarrow \vec{d}_1 \rightarrow \vec{d}_2 \rightarrow \vec{d}_3 \rightarrow \dots$$

the succession of positions ceases to be rectilinear.

A curve appears.

Curvature can therefore be interpreted, within this framework, as a **progressive variation of the direction associated with the successive positions of a single dimensional continuity**.

The dimension is not a straight axis that has been physically bent.

It is constructed from the outset by a succession of position-directions.

If the directions change, the dimension is curved.

4.4. A curved dimension

This construction makes it possible to specify the definition of a curved dimension in directional physics.

A curved dimension can be defined as:

A continuity of position-directions whose orientation varies progressively through space.

Let there be a succession:

$$D_0, D_1, D_2, \dots, D_n$$

with:

$$D_i = (P_i, \vec{d}_i)$$

The overall shape of this dimension then depends on the evolution of the vectors $\vec{d}_i$.

A constant direction produces a rectilinear continuity.

A variable direction produces a curved continuity.

Curvature is therefore not added after the dimension has been defined. **It emerges directly from the way positions and directions are organized.**

4.5. Circles reveal the curve

The representation by circles now makes it possible to understand why they were necessary.

Each circle provides a different level of the construction.

On each one lies a position corresponding to the continuity under study.

When the circles are displayed, the complete structure can be observed:

$$C_0, C_1, C_2, \dots, C_n$$

with their points:

$$P_0, P_1, P_2, \dots, P_n$$

When the circles are then mentally removed and only the corresponding points are kept, what remains is:

$$P_0 \rightarrow P_1 \rightarrow P_2 \rightarrow \dots \rightarrow P_n$$

The resulting curve then reveals the shape of the dimension under study.

The circles thus play the role of an **intermediate structure that makes it possible to construct and locate the points of the dimension.**

4.6. Density modifies geometry

Curvature, however, must not be studied independently of directional density.

In the model, a change in density alters the local organization of the dimensions.

This density can be represented by:

$$\rho(P)$$

Density then becomes a local property depending on the position considered.

When:

$$\rho(P_1) \neq \rho(P_2)$$

the two regions do not necessarily have the same directional organization.

The successive positions and their spacing can therefore be affected by this variation.

The complete representation of the curved dimension must therefore consider:

$$D_i = (P_i, \vec{d}_i, \rho_i)$$

The curve then contains three fundamental pieces of information:

- position;
- direction;
- local density.

In this representation, curvature and densification become two related properties of directional geometry.

4.7. Curvature around a mass

Around a spherical mass, position-directions exist in all orientations.

One must therefore not imagine a single curve.

Take a first orientation:

$$D_0^{(1)} \rightarrow D_1^{(1)} \rightarrow D_2^{(1)} \rightarrow \dots$$

Then a second:

$$D_0^{(2)} \rightarrow D_1^{(2)} \rightarrow D_2^{(2)} \rightarrow \dots$$

Then a third, and so on.

Each series can produce its own curve.

The set of curves surrounding the mass then represents the transformation of the directional structure in all accessible orientations.

In this representation, curvature becomes **omnidirectional**.

This principle avoids the interpretation according to which space would have to bend physically toward a privileged direction.

In this model, curvature exists simultaneously around the mass.

4.8. Curvature between two masses

Now consider two masses such as the Earth and the Moon.

In the three-dimensional construction, each takes part in a structure of spherical surfaces.

In the two-dimensional representation, these surfaces appear as circles.

The corresponding positions can then be followed from the region associated with the Earth to the one associated with the Moon:

$$P_0 \rightarrow P_1 \rightarrow P_2 \rightarrow \dots \rightarrow P_n$$

When the circles progressively change geometry and the directional density varies, the succession of positions can produce a curve.

This curve constitutes a two-dimensional representation of the dimensional continuity between the two regions.

However, it is still only a cross-section.

In the complete construction, this same reasoning can be applied in the various spatial orientations.

We then move from an isolated curve to **a three-dimensional set of curved dimensions.**

4.9. Local curvature and global curvature

This construction also makes it possible to distinguish two levels of description.

Local curvature

Local curvature corresponds to the change observed between successive, very close positions:

$$\vec{d}_i \rightarrow \vec{d}_{i+1}$$

This variation can be very small.

Global curvature

When all the small local variations are considered successively, a global shape appears:

$$D_0 \rightarrow D_1 \rightarrow D_2 \rightarrow \dots \rightarrow D_n$$

A small variation repeated over a large number of positions can thus produce a much larger curve on a large scale.

The global shape of the dimension therefore results from **the accumulation of local directional variations.**

4.10. Measuring directional curvature

This definition also opens the possibility of introducing a measure of curvature specific to the model.

Between two successive positions, one can consider the change of orientation:

$$\Delta\theta_i = \theta_{i+1} - \theta_i$$

If:

$$\Delta\theta_i = 0$$

the local direction remains rectilinear.

If:

$$\Delta\theta_i \neq 0$$

a directional variation exists.

Curvature can then be related to the variation of orientation with respect to the directional distance traveled.

A conceptual formulation can be introduced:

$$\kappa = \frac{\Delta\theta}{\Delta s}$$

where:

- κ represents the local directional curvature;
- $\Delta\theta$ represents the change of orientation;
- Δs represents the interval between two position-directions.

This relation makes it possible to express a simple idea geometrically:

the faster the orientation changes over a small distance, the greater the curvature.

4.11. The dimension as a curve in space

The complete construction now makes it possible to return to the fundamental definition.

A directional dimension is not defined solely by its origin or its initial orientation.

It is defined by the set of position-directions that ensure its continuity:

$$D = \{D_0, D_1, D_2, \dots, D_n\}$$

Its geometry depends on the organization of this set.

If the elements remain aligned, the dimension is straight.

If their orientation varies, the dimension is curved.

If the structure closes back on itself, the dimension can be closed.

The dimension thus becomes a geometric structure capable of taking different forms depending on the organization of its positions and directions.

4.12. From curves to the set

A single curve makes it possible to study one particular dimension.

But it does not yet represent the whole geometry.

Each sphere contains a set of potential positions. Each position can belong to a directional continuity. Each continuity can produce its own curve.

There is therefore a set:

$$\mathcal{C} = \{C_1, C_2, C_3, \dots, C_m\}$$

where each C_i represents a particular curved dimension of the construction.

The complete structure of directional space is then no longer a single curve.

It corresponds to **the set of all curves compatible with the positions, directions, and density of the region considered.**

We thus obtain the complete progression:

sphere → circle → position-direction → curve → set of curves

The next and final chapter will bring these different levels of representation together to show how their combination constitutes, in the model of potential dimensions, a complete representation of the curvature of spacetime.

Chapter 5 — The Set

The previous chapters progressively built the representation of curvature within the framework of potential dimensions. The sphere represents the three-dimensional set of directional possibilities around a position. The circle constitutes a two-dimensional cross-section of this structure. The points make it possible to isolate particular position-directions. Their continuity finally forms a curve corresponding to a curved dimension.

A single curve, however, represents only one particular directional possibility.

To obtain the complete structure, one must bring together **all the positions, all the directions, all the curves, and all the density variations** belonging to the region considered.

This union constitutes **the directional set**.

The curvature of spacetime can then be represented not by a single line or a single surface, but by the collective organization of a very large number of curved directional dimensions.

5.1. From the isolated curve to the set

Consider a first curved dimension:

$$D^{(1)} = \{P_0^{(1)}, P_1^{(1)}, P_2^{(1)}, \dots, P_n^{(1)}\}$$

A second directional continuity can also be constructed:

$$D^{(2)} = \{P_0^{(2)}, P_1^{(2)}, P_2^{(2)}, \dots, P_n^{(2)}\}$$

Then a third:

$$D^{(3)} = \{P_0^{(3)}, P_1^{(3)}, P_2^{(3)}, \dots, P_n^{(3)}\}$$

The process can be repeated for all accessible potential directions.

We then obtain a set:

$$\mathcal{D} = \{D^{(1)}, D^{(2)}, D^{(3)}, \dots, D^{(m)}\}$$

Each element of $\mathcal{D}$ represents a particular position-direction continuity.

The complete directional space corresponds to their collective organization.

5.2. All directions around a point

Around a given position, a direction can be defined along all the distinguishable orientations allowed by the granularity of the model.

A particular curve therefore represents only a fraction of this capacity.

To reconstruct the complete geometry, one must consider simultaneously the continuities starting in all directions.

A first dimension can start toward a given orientation.

A second can start along a slightly different orientation.

A third along yet another orientation.

When the set is reconstructed, the curves progressively fill the directional structure surrounding the position.

Curvature then no longer appears as **a single curve**.

It becomes a **field of curved dimensions**.

5.3. Return to the three-dimensional representation

Circles made it possible to understand the construction in two dimensions, but this representation is only a cross-section.

In the complete geometry, each circle corresponds to the section of a spherical surface.

The succession:

circle → circle → circle

must therefore be placed back into its three-dimensional form:

sphere → sphere → sphere

In the same way, the curve observed in a plane is only one particular dimension among all those that can be constructed around the center.

Rotating the cross-section plane around the mass conceptually makes it possible to recover the other orientations.

One then passes from a family of curves visible in a plane to a volumetric structure in which curved dimensions exist in all orientations.

5.4. The mass at the center of the structure

Let us now consider a spherical mass M .

Around this mass lies a succession of directional surfaces:

$$S_0, S_1, S_2, \dots, S_n$$

Each surface has its potential positions.

Each position can be linked to a corresponding position on a neighboring surface.

The continuities thus obtained form the directional dimensions:

$$D^{(1)}, D^{(2)}, D^{(3)}, \dots$$

If the local structure progressively modifies their orientations, these dimensions become curved.

All of these dimensions then form a complete structure around the mass:

$$\mathcal{D}_M = \bigcup_i D^{(i)}$$

The mass is therefore not represented as an object resting on a surface that sinks.

In this representation, it lies **within a three-dimensional directional geometry that completely surrounds it.**

5.5. A curvature in all directions

This construction leads to a fundamental property of the model.

There is no single direction in which space must curve.

For an ideal spherical mass, the construction can be repeated around the whole body.

A dimension can be studied along a first orientation.

Another along a second.

Then a third.

And so on, until all the distinguishable orientations have been considered.

Curvature therefore becomes omnidirectional.

Each local dimension may exhibit its own directional variation, but all belong to the same geometric set.

Thus, what is globally called **curvature** corresponds, in this representation, to the simultaneous organization of all these directional variations.

5.6. Density as a component of the set

However, the set cannot be defined solely by position and direction.

Directional density must also come into play.

Each local element can therefore be represented by:

$$D_i = (P_i, \vec{d}_i, \rho_i)$$

where:

- P_i represents the position;

- $\vec{d}_i$ represents the direction;
- ρ_i represents the local directional density.

The set then becomes a distribution of positions, directions, and densities.

It can be written conceptually as:

$$\mathcal{D} = \{(P_i, \vec{d}_i, \rho_i)\}$$

The form of directional space thus depends not only on the orientation of the dimensions, but also on how these dimensions are distributed locally.

The densification of space and curvature must therefore be considered jointly in the complete representation.

5.7. Two masses in the same set

Let us now consider the Earth and the Moon.

The Earth participates in a first organization of spherical surfaces.

The Moon participates in a second organization.

But the space between the two bodies is not made up of two completely independent spaces.

The structures belong to the same directional set.

In the three-dimensional representation, the successive surfaces associated with the two regions meet and progressively join.

In a two-dimensional cross-section, these surfaces appear as circles.

The corresponding points of the different circles then make it possible to reconstruct the curves.

These curves finally make it possible to represent the dimensional continuities between the terrestrial and lunar regions.

Thus, the representation passes from two local spherical structures to **a common directional geometry**.

5.8. Continuity between the masses

Between two masses, the different dimensions do not abruptly vanish only to reappear around the second object.

They belong to a single spatial construction.

One can therefore follow a succession of positions:

$$P_0 \rightarrow P_1 \rightarrow P_2 \rightarrow \cdots \rightarrow P_n$$

from a region near the first mass to a region near the second.

The local direction may vary throughout this succession.

The density may also evolve:

$$\rho_0 \rightarrow \rho_1 \rightarrow \rho_2 \rightarrow \cdots \rightarrow \rho_n$$

The geometry between the two masses is then described by the joint evolution of:

position, direction, and density.

It is the totality of these evolutions that determines the directional form of the region.

5.9. Spacetime as a structure of curved dimensions

The construction can now be generalized.

Each region of space possesses a set of potential dimensions.

Each dimension associates a position with a direction.

Successive positions define continuities.

Variations in direction produce curves.

Variations in density modify the local organization of these continuities.

The set then forms a global directional structure.

The curvature of spacetime can thus be represented, in this model, as:

the collective organization of the position-direction dimensions and of their density in a given region.

Curvature is no longer reduced to the image of a deformed surface.

It corresponds to a transformation of the directional structure itself.

5.10. From the local to the global

An important property then appears.

A very small local variation in direction may seem negligible:

$$\Delta\theta_i \approx 0$$

But when a very large number of successive variations are added together, their global effect can become significant.

A macroscopic curve can therefore emerge from a succession of extremely small directional transformations.

The same principle applies to the complete set.

The global geometry around a mass emerges from the organization of a very large number of local transformations.

We thus obtain two complementary levels:

local level:

$$P_i, \vec{d}_i, \rho_i$$

global level:

$$\mathcal{D} = \{D^{(1)}, D^{(2)}, \dots, D^{(m)}\}$$

The first describes the elementary structure.

The second describes the geometry produced by their totality.

5.11. A complete representation

The five elements constructed in this document can now be brought together.

Potential dimensions define the accessible directional possibilities.

Spheres represent these possibilities in all orientations around a center.

Circles provide a two-dimensional cross-section of the construction.

Curves appear when the corresponding position-directions are connected.

The set brings together all the curves and all the densities in a single three-dimensional structure.

The complete progression therefore becomes:

potential dimensions → spheres → circles → curves → directional set

Each step is not independent of the previous one.

It constitutes a change of scale or of representation that progressively allows the complete geometry to be reconstructed.

5.12. Concise definition of the directional curvature of spacetime

From this construction, a general definition can be proposed:

In the model of potential dimensions, the curvature of spacetime corresponds to the non-rectilinear organization of the set of position-direction continuities constituting a region of space, together with the local variations of their directional density.

This definition contains the elements developed throughout the document:

position + direction + continuity + density + set

An individual dimension may be straight or curved.

A region contains several dimensions.

In the model, a mass influences the local organization of these dimensions.

And the set of these transformations constitutes the directional geometry of the region.

5.13. Synthesis of the construction

The representation can finally be understood in both directions.

Starting from the complete structure:

set → curves → circles → spheres → potential dimensions

one progressively decomposes the geometry in order to understand its constituents.

Starting from the fundamental element:

potential dimension → sphere → circle → curve → set

one progressively reconstructs the complete geometry.

These two readings lead to the same principle:

Global curvature emerges from the local organization of positions and directions.

Curvature is therefore not external to the dimensions.

In this framework, it is a property of their collective organization.

Chapter Conclusion

The set constitutes the last step necessary to move from an individual dimension to a global representation of curvature.

A sphere alone represents the directions of one level.

A circle alone represents a cross-section.

A curve alone represents a directional continuity.

But when all these continuities are brought together, the complete geometry appears.

In the framework developed here, space thus becomes a geometric network of **position-directions whose orientation and density can vary locally**.

A mass is no longer represented as an object pressing down on a sheet. It belongs to a three-dimensional structure in which the surrounding dimensions can be studied simultaneously in all orientations.

The representation of curvature thus becomes volumetric, directional, and progressive.

The sphere gives the possibilities.

The circle reveals the cross-section.

The curve reveals the dimension.

The set reveals the geometry.